

Semileptonic Form Factors

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I report the current status of the heavy-light decay constants, the bag parameters and the semileptonic form factors. I compare the heavy-light decay constants with Wilson-Wilson and clover-clover fermions. Systematic errors such as scale setting and renormalization factors are also discussed. $1/M$ dependences for the heavy-light semi-leptonic form factors near $q^2 = q_m^2 ax$ with clover-clover and NRQCD-Wilson fermions found to be small.

1. Introduction

Calculations of the B meson decay constants, the bag parameters and the semileptonic form factors have been attracting the attention of lattice community over the past several years, since those are indispensable ingredients for the determination of V_{CKM} and then the test of the standard model as well as the study of CP violation and the quark mass generation. I review the update in Lattice QCD made in this year.

For the formalism of the heavy quark on the lattice the Fermilab group [1] proposed a systematic method to treat the error associated with the large heavy quark mass in the relativistic formalism. Then the large scale calculations of Fermilab, JLQCD and MILC collaborations at several β values using the Wilson or clover action are giving a reliable prediction for f_B . There has also been a considerable development in the calculation of f_B using the NRQCD formalism, which was reviewed by Ali-Khan in this conference[2].

New calculations of the semi-leptonic decay form factors $B \rightarrow \pi/\rho$ have become available by Hiroshima and JLQCD collaborations using the NRQCD and the clover actions for heavy quark. Their simulations treat the b -quark directly without extrapolating from the charm quark mass region, and the heavy quark mass dependence of the form factors are clarified.

2. Heavy Quark on the Lattice

In currently accessible simulation, the bottom and charm quark masses in lattice unit are large: $m_b a = 1 \sim 4$, $m_c a = 0.4 \sim 1$. Since the brute force extrapolation to the continuum limit is not realistic, one has to formulate an effective theory in which there is no large systematic uncertainty of $O(m_Q a)$. For Wilson and clover action, Fermilab interpretation is such an example[1]. In their formalism the relativistic heavy quark is interpreted as an effective action in the heavy quark rest frame, which is only valid in the small spatial momentum regime.

In order for high precision calculation of the matrix elements, it is important to understand the source of systematic errors. In the following, I discuss the expected size of the systematic errors in the calculations of the matrix elements of the heavy-light mesons with the Wilson/clover fermions.

Besides quenching errors, some of the largest sources of errors are perturbative and the lattice discretization errors from the action and the operators.

First let us discuss the light quark action. Since the lattice cutoff a^{-1} and the typical momentum scale of the quark (and gluons) p are the only dimensionful quantity, the above errors can be expanded in powers of ap and α_s . As is well known, the leading error for the Wilson light quark comes from the lack of the clover term, which is $O(ap)$. On the other hand, the leading errors for the clover light quark are $O((ap)^2)$,

$O(\alpha_s^2)$, and $O(\alpha_s ap)$. There exists heavy-light simulations with $c_{sw} = 1/u_0^3(1 + 0.199\alpha_s)$ [3] at one-loop level by JLQCD, in which case the leading errors of the action are $O((ap)^2)$ and $O(\alpha_s^2 ap)$ only.

The situation is similar for heavy quark action. The errors can again be expanded in powers of α_s and ap , except that the coefficients which appear in the expansion are now some functions of am_0 where m_0 is the bare quark mass. In the Fermilab interpretation of the Wilson/clover quark, the small momentum expansion of the effective Hamiltonian reads[1] as,

$$\begin{aligned} H = & -\left(\frac{1}{2m_2a} + b^{(1)}\alpha_s + \dots\right)\vec{D}^2 \\ & -\left(\frac{1}{2m_Ba} + b_B^{(1)}\alpha_s + \dots\right) \cdot i\vec{\Sigma} \cdot \vec{B} \\ & + \text{higher order}, \end{aligned} \quad (1)$$

with m_2a , m_Ba , $b^{(1)}$, $b_B^{(1)}$, ... are functions of m_0a .

The tree level coefficients in the above expression are described as follows for Wilson/clover action,

$$\begin{aligned} \frac{1}{2m_2a} &= \frac{1}{m_0a(2+m_0a)} + \frac{1}{1+m_0a} \\ \frac{1}{2m_Ba} &= \frac{1}{m_0a(2+m_0a)} + \frac{c_{sw}}{1+m_0a}. \end{aligned} \quad (2)$$

The leading error in the Wilson heavy quark action arises in the spin-magnetic coupling, as $1/2m_Ba$ differs from $1/2m_2a$ by $1/(1+m_0a)$. Thus the size of the error is estimated to be of $O(p/m_0)$, which may relatively be smaller than the error from the light quark action. For clover heavy quark action, the error from the spin-magnetic coupling arises at one-loop level, which is $O(\alpha_s ap)$ or $(\alpha_s p/m_0)$. Therefore, the leading errors in clover heavy quark are $O(\alpha_s ap)$, $O(\alpha_s p/m_0)$, $O((ap)^2)$ and $O((p/m_0)^2)$. So far there exists no calculation of c_{sw} at one-loop for finite quark mass, the knowledge of which would be useful in removing the error of $O(\alpha_s ap)$ and $O(\alpha_s p/m_0)$.

Next let us consider the error from the current operator by taking the example of the heavy-light axial-vector current. The heavy-light current operator has the following form in the Fermilab in-

terpretation:

$$\begin{aligned} A_4 = & d_0 \overline{\psi}_q \gamma_5 \gamma_4 \psi_Q \\ & + d_1 \overline{\psi}_q \gamma_5 \gamma_4 \vec{\gamma} \cdot \vec{D} \psi_Q \\ & + d_2 \vec{D} \overline{\psi}_q \cdot \vec{\gamma} \gamma_5 \gamma_4 \psi_Q, \end{aligned}$$

where

$$\begin{aligned} d_0 &= 1 + d_0^{(1)}\alpha_s + \dots \\ d_1 &= -\frac{1}{2m'} + d_1^{(1)}\alpha_s + \dots \\ d_2 &= d_2^{(1)}\alpha_s + \dots, \end{aligned}$$

and

$$-\frac{1}{2m'a} = -\frac{1}{2m_2a} + \frac{m_0a^2}{2(1+m_0a)(2+m_0a)} \quad (3)$$

for both Wilson and clover actions. The second term in the right hand side of this equation causes $O(ap)$ error formally, but in practice, it is negligibly small because of the small coefficient $m_0a/(2(1+m_0a)(2+m_0a))$.

Recently, the one-loop correction for the leading term $d_0^{(1)}$ was calculated fully incorporating the heavy quark mass dependence[4]. Therefore, for both Wilson and clover actions, the leading errors are from the higher derivative terms, which are of $O(\alpha_s ap)$ and $O(\alpha_s p/m_0)$ in addition to the $O(\alpha^2)$ and $O((ap)^2)$ errors for both the Wilson and clover actions. For further reductions of the errors, calculation of the one-loop corrections $d_1^{(1)}$ and $d_2^{(1)}$ is necessary. The NRQCD group performed such calculation for NRQCD heavy quark[5]. They find the effect of operator mixing reduces f_B by about 10% at $\beta=6.0$. It would be interesting to see whether it is also the case for clover heavy quark case.

To summarize, the Wilson heavy-light system has leading errors of $O(ap)$ and $O(p/m)$, while the clover heavy-light system has leading errors of $O(\alpha_s^2)$, $O(\alpha_s ap)$, $O(\alpha_s p/m)$ and $O((pa)^2)$, $O((p/m)^2)$. Future one-loop renormalization with massive fermion for both the action and the current remove one of the leading errors, which is $O(\alpha_s ap)$ and $O(\alpha_s p/m)$.

3. Lattice Results

In this section, we review the numerical simulation results for the decay constant and the form

factors. We discuss whether the picture for the systematic errors in the previous section explains the lattice spacing dependence of the actual data or not, which is of particular importance in extrapolating to the continuum limit.

3.1. Decay Constants

Table 3.1 shows results on heavy-light decay constants by various groups using Wilson/clover fermions.

MILC collaboration uses Wilson fermion in the Fermilab formalism, in the quenched approximation with scales set from f_π and m_ρ . They also perform simulations with dynamical configurations for $\beta = 5.445, 5.5, 5.6$. They get their best preliminary results with the scale from f_π as shown in Table 3.1. The systematic errors from quenching are estimated by comparing the result with scales set from m_ρ and f_π , and also the result with $n_f = 0$ and $n_f = 2$. They also study the chiral extrapolation error by comparing the results with the linear and quadratic fits in the chiral extrapolation. They use the local axial vector current in KLM normalization, with Z_A from one-loop perturbation theory in the massless limit. The renormalization scale for α_v is $q^* = 2.32/a$ [6].

JLQCD

collaboration uses Wilson/clover fermions in the Fermilab formalism on three different lattices in the quenched approximation. They obtain their preliminary results with scales set from f_π , m_ρ and the string tension with f_{m_ρ} for best results. They use the local axial vector current in KLM normalization with Z_A from one-loop perturbation theory, in which the heavy quark mass effect is taken into account[4]. The renormalization scale is $q^* = 1/a$.

APE collaboration uses both Wilson/clover fermions with $\beta = 6.0, 6.2$ in the quenched approximation[7]. They compute the heavy-light decay constants in the charm quark mass range and extrapolates them in $1/M$ to bottom quark mass range. They first compute the ratio $R \equiv f_{PS}/f_\pi$ with scales set from f_π , m_ρ , the string tension and f_{K^*} , then multiply f_π^{exp} . They find consistent results for all these four scale settings. They use local current both in KLM and in standard

normalization. For setting the scale from f_π using lattice results they use Z_A determined nonperturbatively using chiral Ward identities. [8,9]

UKQCD collaboration uses clover fermion in the quenched approximation. They use the rotated axial vector current with Z_A from one-loop perturbation theory. They obtain their best results with scales set from m_ρ [10,11].

Fermilab group performed calculations of the heavy-light decay constant with clover fermion in Fermilab formalism in the quenched approximation. They use the tree-level improved current with Z_A obtain from one loop perturbation theory in the massless limit. The scale is set from f_K . They first extrapolate the $f_B M_B^{1/2}$ linearly to the continuum then obtain the decay constant in order to avoid introducing unnecessary systematic errors from the heavy meson mass.

Figures 1 and 2 show the lattice spacing dependences of f_B by JLQCD and MILC collaboration using Wilson-Wilson and clover-clover fermions, whose scale are determined from f_π and m_ρ respectively. Figure 3 shows the data with scales from the string tension[12]. Also, Figure 4 shows the lattice spacing dependence of the $f_B M_B^{1/2}$ by Fermilab Group with scales set from f_π .

The naive error estimate in the previous section suggests that the Wilson data should scale linearly in the lattice spacing a with a rather steep slope. Taking typical momentum p as $0.3 \sim 0.5 GeV$, the data at $a = 0.5 GeV^{-1}$ can typically have 15-25% $O(a)$ error.

On the other hand, since the clover data should have $O((ap)^2)$, $O(\alpha_s ap)$ and $O(\alpha_s^2)$, one expects 5% errors from each contribution, thus the a dependence for the clover data should be much smaller than that for Wilson data.

This is indeed the case for data with scales from m_ρ and string tension. For example, Figure 2 shows that the clover f_B has very small a dependence, while the Wilson f_B has a large a dependence. Moreover, the clover and the Wilson data seem to have continuum limit which are consistent with each other. It seems that the MILC Wilson data are slightly higher than JLQCD Wilson data by a few %, which will be discussed later.

Now the situation for f_B with scales from f_π

Table 1

Decay constant calculation from various groups using Wilson/clover fermions. The decay constants are in MeV units. The errors of the MILC result are statistical, systematic errors except quenching and quenching error. For JLQCD, the errors are statistical, systematic errors from chiral extrapolation and so on, and the systematics from choosing the scale setting. The errors of Fermilab group are statistical, systematic errors from scale setting, and other systematic errors respectively. The results of MILC, JLQCD and Fermilab are preliminary.

Group	MILC [13]	JLQCD [14]	APE [7]	UKQCD [10,16]	FNAL [17]
Action	Wilson	Wilson clover	Wilson clover	clover	clover
β	5.72 ~ 6.52	5.9, 6.1, 6.3	6.0, 6.2	6.0, 6.2	5.7, 6.1, 6.3
a^{-1} from f_π		m_ρ	$f_\pi, m_\rho, \sigma, f_{K^*}$	m_ρ	f_K
f_B	153(10)($^{36}_{13}$)($^{13}_0$)	163(9)(8)(11)	180(32)	160(6)($^{53}_{19}$)	156($^{13}_{14}$)($^{29}_0$)($^{0}_{20}$)($^{0}_5$)($^{2}_0$)
f_{B_s}	164(9)($^{47}_{13}$)($^{16}_0$)	175(9)(9)(13)	205(35)		177($^{11}_{12}$)($^{39}_0$)($^{13}_{12}$)
f_D	186(10)($^{27}_{13}$)($^{9}_0$)	184(9)(9)(12)	221(17)	185($^{4}_3$)($^{42}_7$)	183($^{12}_{13}$)($^{41}_0$)($^{9}_{25}$)
f_{D_s}	199(8)($^{40}_{11}$)($^{10}_0$)	203(9)(10)(14)	237(16)		229($^{10}_{11}$)($^{51}_0$)($^{3}_{19}$)
f_{B_s}/f_B	1.10(2)($^{5}_3$)($^{3}_2$)		1.14(8)	1.22($^{4}_3$)	1.17(5)(10)(3)
f_{D_s}/f_D	1.09(2)($^{5}_1$)($^{2}_0$)		1.07(4)	1.18($^{2}_2$)	1.22(5)(0)(8)

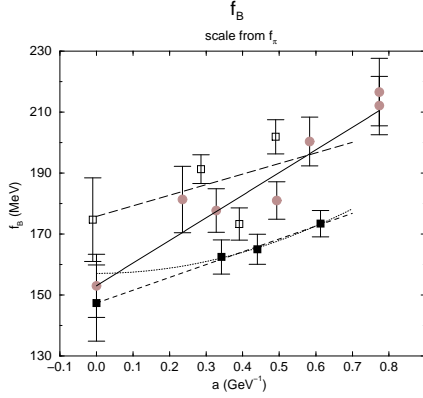


Figure 1. Continuum extrapolation of f_B from MILC Wilson (diamond), JLQCD Wilson (open square) and JLQCD clover (filled square) using f_π for the lattice scale.

is not clear. The linear slope of Wilson f_B is entirely different between JLQCD and MILC collaboration. Also within JLQCD data the continuum limits from Wilson and Clover are not in agreement within error.

Here, three questions arise:

1. Are the renormalization factors Z_A consistent between two groups?

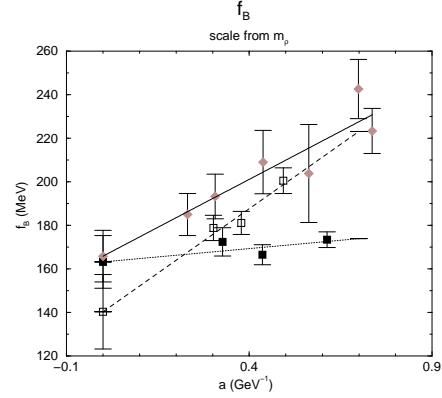


Figure 2. Continuum extrapolation of f_B from MILC Wilson (diamond), JLQCD Wilson (open square) and JLQCD clover (filled square) using m_ρ for the lattice scale.

2. Are the raw data consistent?
3. Are there any problems in setting the scale, especially from f_π ?

In the following, we will consider these problems in detail.

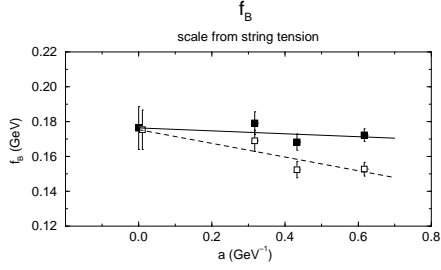


Figure 3. Continuum extrapolation of f_B . Filled square and open square are JLQCD Wilson and clover results using string tension for the lattice scale.

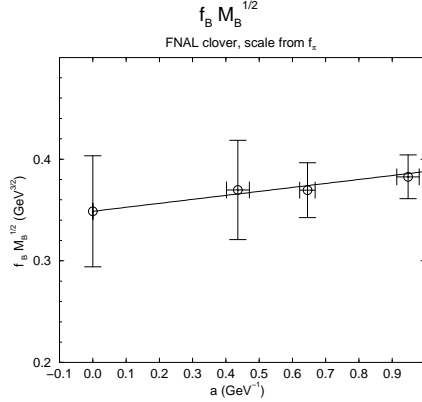


Figure 4. Continuum extrapolation of $f_B M_B^{1/2}$ by Fermilab group. The scale is set from f_π .

3.1.1. Renormalization factor

As was mentioned in the introduction, one-loop renormalization factor Z_A for the heavy-light axial vector current has been calculated as a function of the heavy quark mass[4,18]. Figure 5 shows the mass dependence of the one-loop coefficient of Z_A . It is found that although there is indeed heavy quark mass dependences in Z_A , for $am_0 \sim 1 - 3$, the difference between the Z_A for heavy-light axial current and that for the massless light-light axial current is around 2-3 percent for $g^2 \sim 2 - 3$. MILC collaboration uses Z_A in massless limit even for the heavy-light current,

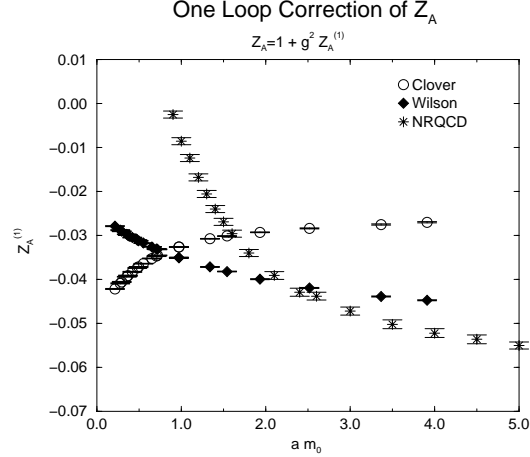


Figure 5. One-loop coefficient of Z_A for Wilson (circles), clover (diamonds) and NRQCD (stars) as a function of the bare heavy quark mass.

while JLQCD uses slightly smaller Z_A with mass dependence included. This explains part of the discrepancy between JLQCD and MILC.

It should also be noted that the scale q^* of the coupling α_v differs between JLQCD and MILC, which are $q^* = 1/a$ and $q^* = 2.32/a$ [6] respectively. Using the same q^* makes the results come closer again by a few percent.

3.1.2. Comparison of the raw data

We compare raw data of $f_P \sqrt{M_P}$ from MILC and JLQCD Wilson. Since MILC and JLQCD has a slightly different analyses, it is not easy to see whether the two groups are consistent with each other from their quoted values. In Figure 6, we give a plot of $f_P \sqrt{M_P}$ at $\beta = 6.3$ where the data are available from both groups. The JLQCD's raw data are re-analyzed in the same way as MILC so that they have the same renormalization factor Z_A with $q^* = 2.32/a$.

We find that two results are in perfect agreement with each other.

3.1.3. Scale setting

The last issue is the scale setting. Since accuracy required in the lattice calculation of f_B is around 10%, the scale a^{-1} used by various groups

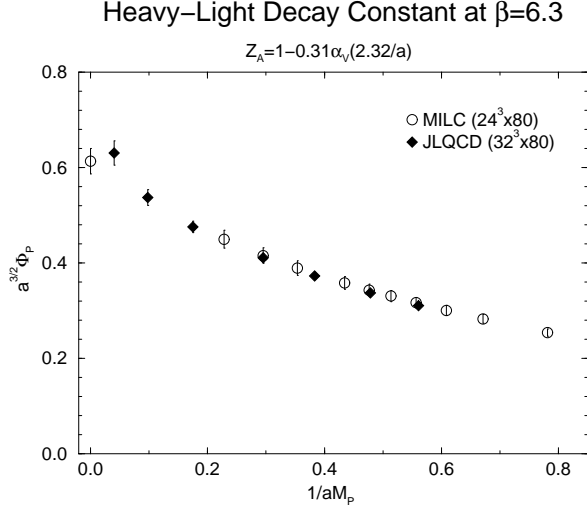


Figure 6. Direct comparison of JLQCD and MILC.

are in fact important. Of course, there should be differences between the scales a^{-1} from different inputs, which are nothing but the queching effects. However, any disagreement in the scales a^{-1} from the same inputs due to the fitting procedure etc. can be the source of systematic error which has to be removed for the high precision calculation of f_B . Another problem is the consistency of the results in the continuum limits from different actions. Although the scale from the same input can differ for Wilson and clover for finite cutoff, the continuum limit should agree. If they differ, there should be some systematic error such as fitting procedure, renormalization, or the way to take the continuum limit.

- a^{-1} from m_ρ

Figure 7 shows the ratio of the scale a^{-1} from m_ρ to that from the string tension (obtained by Balli-Schilling[12]) for Wilson and clover fermions.

It can be seen that the $a^{-1}(m_\rho)$ from JLQCD, CPPAX, MILC and APE Wilson data agree with each other over the whole range. The clover JLQCD, APE and UKQCD data have consistently smaller values but the continuum limit agrees with

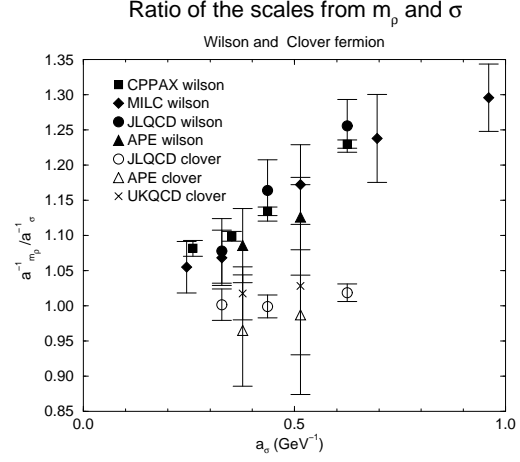


Figure 7. a^{-1} from m_ρ . Filled circle is the JLQCD Wilson data, filled diamond is the MILC Wilson data, square is the CPPAX Wilson data, Filled triangle is the APE Wilson data. Open circles are JLQCD clover data, open triangles are APE clover and crosses are UKQCD clover data.

that for Wilson within statistical errors.

- a^{-1} from f_π

Figure 8 shows the ratio of the scale a^{-1} from f_π to that from the string tension for Wilson and clover fermions.

The result from JLQCD Wilson and CP-PACS Wilson are in agreement, whereas MILC results with linear chiral extrapolation are more than 10% smaller than those by JLQCD and CP-PACS over almost the whole range. MILC results with quadratic chiral extrapolation lie in between. It should be noted that the perturbative renormalization scale q^* used for α_v differs between JLQCD-CP-PACS and MILC, which are $q^* = 1/a$ and $q^* = 2.32/a$ respectively. Using the same q^* makes the results come closer, but by only a few percent. The origin for the rest discrepancy is not clear: it may either come from the uncertainty of the linear extrapolation or some other effects such as finite volume ef-

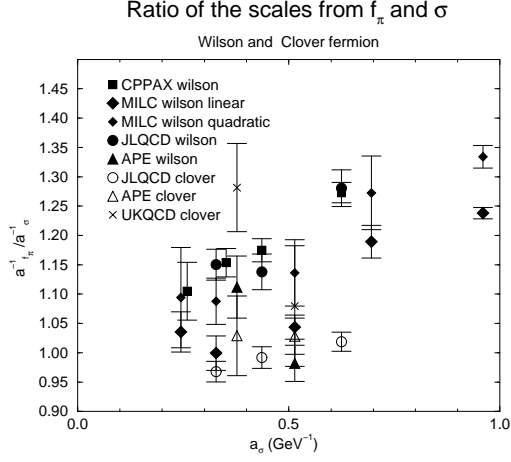


Figure 8. a^{-1} from f_π . Filled circle is the JLQCD Wilson data, filled large(small) diamonds are the MILC Wilson data with linear(quadratic) chiral extrapolation, square is the CPPAX Wilson data, Filled triangle is the APE Wilson data. Open circles are JLQCD clover data, open triangles are APE clover and crosses are UKQCD clover data.

fects. APE wilson and UKQCD clover data have large a dependences, while APE clover data are consistent with JLQCD data.

Therefore, using Wilson fermions JLQCD and MILC collaborations obtain consistent scale determined from m_ρ , which seem to have consistent continuum limit with that by JLQCD using clover fermions.

For the scale determined from f_π , the results with Wilson fermions from JLQCD and MILC collaborations do not agree. The continuum limit of the clover results from JLQCD are consistent with that by MILC Wilson data but not with JLQCD Wilson data.

3.1.4. Effects of Quenching

Let us now consider the systematic error from the quenched approximation. MILC collaboration estimated effects of quenching in two ways: (1) by comparing the a smallest dynamical lattice, with the quenched results interpolated to the same lattice spacing and (2) redoing the calcula-

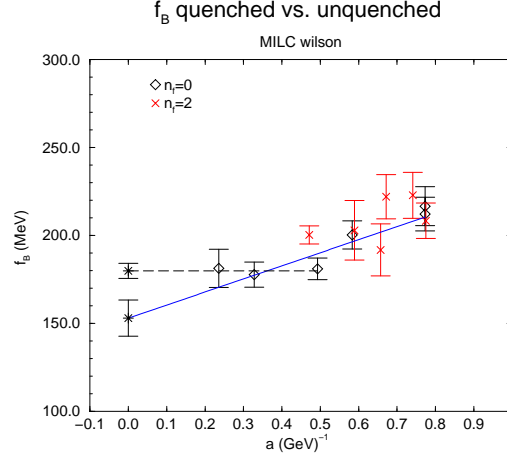


Figure 9. f_B for quenched and unquenched Wilson by MILC collaboration.

tion switching from the f_π scale to the m_ρ scale. They take the biggest deviation from the central value as the error estimate. In Figs.9,10 quenched results and unquenched results of f_B and f_{B_s}/f_B are compared.

3.1.5. Summary of f_B

The study of the previous section shows that Wilson data are essentially in agreement. What makes the result different is the choice of the renormalization factor and the scale. We observe that m_ρ is a reliable quantity to set the scale, while there are still uncertainties for f_π . The final result of MILC Wilson, JLQCD Wilson/clover, UKQCD clover, FNAL clover for f_B are in good agreements. The central value of APE result is higher than others but within errors. The global value of f_B in the quenched approximation I quote is JLQCD result is:

$$f_B = 163(9)(8)(11)\text{MeV}. \quad (4)$$

For the quenching error, I quote the error estimated by MILC collaboration which is slightly less than 10 %.

3.2. Bag Parameters

The Bag parameter is calculated using static heavy and clover light by UKQCD for $\beta = 6.2$ [11],

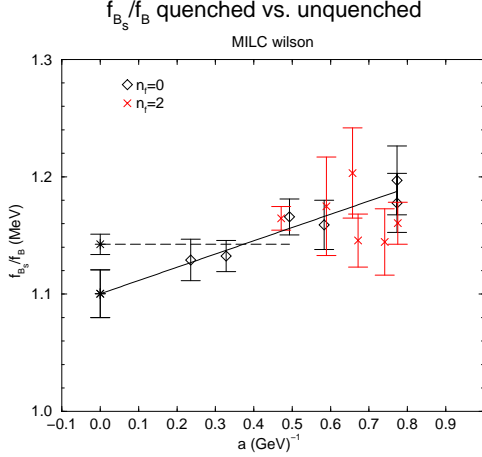


Figure 10. f_{B_s}/f_B for quenched and unquenched Wilson by MILC collaboration .

Table 2
Results for the Bag parameters. The results from MILC collaboration are preliminary.

Group	β	$B_B(m_b)$	\hat{B}_B^{NLO}
MILC[20]	5.85($n_f = 0$)	0.94(4)	
	5.5($n_f = 2$)	0.87(2)	~ 1.4
	5.6($n_f = 2$)	0.94(3)	
G+M [19]	6.0	0.63(4)	1.00(6)
		0.73(4)	1.16(6)
UKQCD[11]	6.2	0.69(4)	1.10($^{(5)}_6$)($^{(3)}_2$)

by Gimenez and Martinelli [19] for $\beta = 6.0$ in the quenched approximation, and by MILC [20] for $\beta = 5.85$ in the quenched approximation, and for $\beta = 5.5, 5.6$ with $n_f = 2$.

3.3. Semileptonic Form Factors

In spite of many years of effort [21] to compute the semileptonic form factors, heavy-light to light-light such as $B \rightarrow \pi, \rho$ is still a big challenge to lattice QCD. The reason for the difficulty is that the form factors at lower q^2 has large statistical errors since the noise of the three point functions grows as $\sim \exp(E_{\pi, \rho} t)$. Moreover, the

systematic error due to the finite lattice spacing which is of $O(ap)$ for Wilson fermion and the $O((ap)^2)$ and $O(\alpha a)$ for clover fermion makes it hard to obtain the correct q^2 dependence. This is different from the situation in D meson decay [22, 24, 23, 27, 25–28], in which case the highest momentum of the final state meson is less than 1 GeV so that the lattice simulation can cover the whole kinematically allowed range without encountering the above difficulties.

Table 3.3 shows the list of the simulations on heavy-light to light-light form factors. In Earlier works simulations with heavy quark in the charm quark mass range were done mostly to study the D meson decay. However, the results can be extrapolated in $1/M$ to the B meson decay. There are two approaches to extrapolate the results, both of which uses $1/M$ scaling from the charm quark mass range for the heavy quark mass. The first approach, taken by UKQCD, for example, is to scale the form factors near q_{max}^2 assuming the following scaling laws predicted from the heavy quark effective theory:

$$\begin{aligned} f^0, A_1, T_2 &\sim M^{-1/2} \\ f^+, V, A_2, T_1 &\sim M^{1/2} \end{aligned}$$

In this approach, the range of q^2 is limited to the range near q_{max}^2 . In order to know the form factors for smaller q^2 , one has to extrapolate the result by fitting the form factors to some model such as the pole model.

Another approach, taken by the Wuppertal group, for example, is to obtain the form factors at $q^2 = 0$ for charm quark mass range, which can reliably be done by interpolation. The form factors are then scaled in $1/M$. The second method requires the knowledge of the scaling law at $q^2 = 0$, which heavy quark effective theory cannot predict.

If one assumes the pole model or its generalization, these two approaches are related to each other since the form factor ansatz in the model relates the $1/M$ scaling behavior near q_{max}^2 and at $q^2 = 0$.

Fermilab group [32] has studied the $1/M$ dependence of the form factors f^+, f^0 with heavy quark mass at the static, the bottom and the charm quark mass and with light quark mass

Table 3

Simulations of heavy-light to light-light semileptonic decay form factors by various groups. The results from JLQCD collaboration and Hiroshima collaboration are preliminary.

Group	β	Heavy(Light) Actions	m_Q	m_q
JLQCD[29]	5.9	clover(clover)	$m_c \sim m_b$	chiral limit
Hiroshima [30]	5.8	NRQCD(Wilson)	$m_c \sim m_b$	chiral limit
Wuppertal[25]	6.3	Wilson(Wilson)	$\sim m_c$	chiral limit
UKQCD[15,31]	6.2	clover(clover)	$\sim m_c$	$\sim m_s$
FNAL[32]	5.9	clover(clover)	m_c, m_b, ∞	$\sim m_s$
APE[23]	6.0	clover(clover)	$\sim m_c$	chiral limit
LANL[28]	6.0	Wilson(Wilson)	$\sim m_c$	chiral limit
ELC[24]	6.4	Wilson(Wilson)	$\sim m_c$	chiral limit

at the strange quark. Recently, JLQCD collaboration[29] and Hiroshima collaboration[30] performed calculations of the semileptonic form factors near q_{max}^2 using clover heavy - clover light and NRQCD heavy - Wilson light. Both groups treat the heavy quark action as the nonrelativistic effective action, hence the simulations at the b quark mass range are possible.

In the following, we discuss the scaling behavior in $1/M$.

3.3.1. $1/M$ scaling near q_{max}^2

Fig.11 shows M dependence of $f^0(q_{max}^2)M^{1/2}$. It is found that the $1/M$ scaling behavior is consistent with the prediction of the heavy quark effective theory and that the $1/M$ correction is small. JLQCD and Hiroshima gives consistent result. They are also consistent with the result of UKQCD which is obtained by scaling the form factor near the charm mass range up to the bottom quark mass.

Fig.12 is the plot of the M dependence of $f^+(q^2)/M^{1/2}$ with the pion momentum $k_\pi = \frac{2\pi}{L} \cdot (1, 0, 0)$. Again the data confirms the $1/M$ scaling behavior predicted by the heavy quark effective theory.

In Fig.13 $A_1(q_{max}^2)M^{1/2}$ is plotted against $1/M$. The $1/M$ scaling behavior is again consistent with the heavy quark effective theory. There is a significant discrepancy of the JLQCD data with the UKQCD data. JLQCD group found that the $A_1(q_{max}^2)$ decreases rather rapidly as the light quark mass approaches to the chiral limit. Since the result of UKQCD is obtained not in the chi-

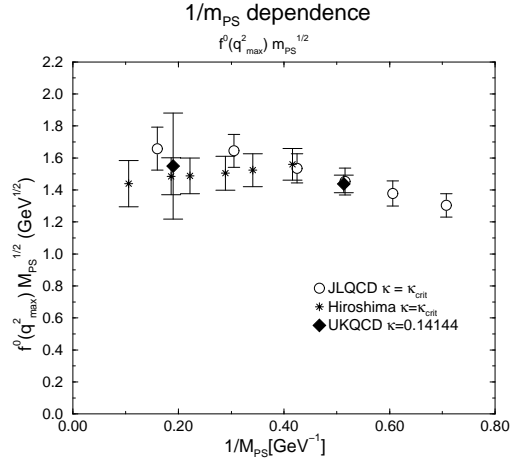


Figure 11. $f^0(q_{max}^2)M^{1/2}$ from various groups

ral limit but with the strange light quark, the discrepancy may be explained by the difference in the light quark mass.

There is another problem in the chiral limit. Soft pion theorem predicts the relation $f^0(q_{max}^2) = f_B/f_\pi$ [33–35]. Fig.14 gives plots of $f_B M^{1/2}/f_\pi$ and $f^0(q_{max}^2)$ by Hiroshima Group. The two quantity have a large discrepancy in the heavy quark mass limit. JLQCD group also finds similar phenomena. Possible reasons are the breaking of chiral symmetry on the lattice, the renormalization, the systematic uncertainty of the chiral extrapolation, or the breakdown of the soft pion theorem itself in the

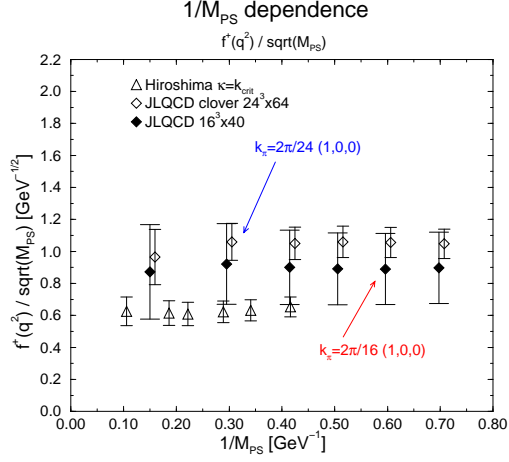


Figure 12. $f^+(q^2)/\sqrt{M_{PS}}$ in the chiral limit as a function of $1/M_{PS}$. Open diamonds correspond to the results on a $16^3 \times 40$ lattice and filled diamonds on a $24^3 \times 64$ lattice. The spatial momentum of B meson is zero and the pion is moved with the smallest momentum in lattice units. Triangles are the results from Hiroshima NRQCD + Wilson results

semileptonic decay. This needs further study.

The q^2 dependences of the form factors f^0, f^+ for B meson with strange light quark are shown in Fig.15. The Hiroshima Groups result and the UKQCD result extrapolated from the charm quark mass range are consistent. In the chiral limit, the statistical error of the form factors with nonzero recoil becomes very large so that the pion signal with only zero and the smallest unit of momentum can be observed. Clearly to obtain the form factors in the chiral limit with pion momentum of 1GeV , one needs statistics of a few hundred configurations.

For strange light quark mass, the statistics is good enough to extract q^2 dependence upto $k_\pi \sim 1\text{GeV}$, in fact $B \rightarrow \pi + l + \bar{\nu}$ semileptonic decay rate clearly exhibits the pion spectrum, however, the present level of the systematic error in the q^2 dependence is $O(ap)$ for Wilson fermion which gives a large uncertainty in the decay rate. Therefore the use of improved actions and calcu-

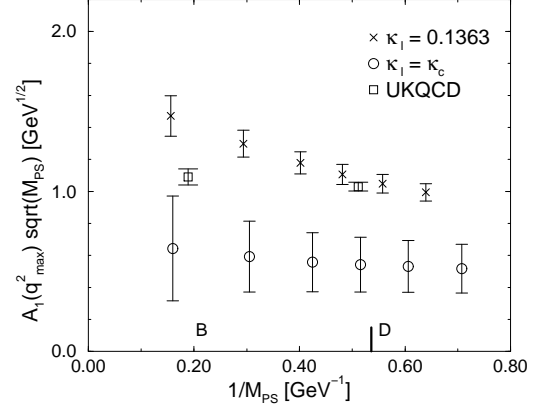


Figure 13. Squares are the data of $A_1(q^2_{max})\sqrt{M_{PS}}$ from UKQCD which is extrapolated $1/M$ with the light quark at $k_l = 0.14144$, and crosses and circles are those of JLQCD on a $16^3 \times 40$ lattice at $\kappa = 0.1363$ and κ_c as a function of $1/M_{PS}$.

lations on several lattices with larger β are required as was the case for the heavy-light decay constant.

3.3.2. $1/M$ scaling at $q^2 = 0$

According to the prediction of the light cone sum rule (LCSR)[36], all the form factors at $q^2 = 0$ scales as $M^{-3/2}$. Using pole-type models with the $1/M$ scaling behavior which are consistent both with HQET and LCSR, UKQCD group fitted their lattice data to the form factors for $B \rightarrow \pi, \rho$ to obtain the form factors at q^2 .

On the other hand, Wuppertal group scaled the form factors at $q^2 = 0$ obtained from simulation at the charm quark range without necessarily assuming LCSR prediction. Instead, they tried three different ansätze for the $1/M$ scaling, which are $F(0) \sim M^{-1}, M^{-3/2}$ and $M^{-1/2}$. The present statistics does not distinguish the three different fits so that they get consistent results. The situation is not yet clear with the present statistics. Further study is need to test the scaling behavior at q^2 . The results for the semileptonic form

Table 4
Semileptonic decay form factors at $q^2 = 0$.

$B \rightarrow \pi/\rho$				
Group	$f(0)$	$V(0)$	$A_1(0)$	$A_2(0)$
Wuppertal[25]	0.43(19)	0.65(15)	0.28(3)	0.46(23)
UKQCD[22]	0.27(11)	0.35($\frac{6}{5}$)	0.27($\frac{5}{4}$)	0.26($\frac{3}{3}$)
APE a[23]	0.29(6)	0.45(22)	0.29(16)	0.24(56)
APE b[23]	0.35(8)	0.53(31)	0.24(12)	0.27(80)
ELC a[24]	0.26(12)(4)	0.34(10)	0.25(12)	0.25(6)
ELC b[24]	0.30(14)(5)	0.37(11)	0.27(5)	0.49(21)(5)
$D \rightarrow K/K^*$				
Group	$f(0)$	$V(0)$	$A_1(0)$	$A_2(0)$
Wuppertal[25]	0.78(5)	1.27(16)	0.67(4)	0.67(13)
LMMS[26]	0.63(8)	0.86(10)	0.53(3)	0.19(21)
BKS[27]	0.90(8)(21)	1.43(45)(49)	0.83(14)(28)	0.59(14)(24)
APE[23]	0.77(4)	1.16(16)	0.61(5)	0.49(34)
ELC[24]	0.60(15)(7)	0.86(24)	0.64(16)	0.40(28)(4)
LANL[28]	0.73(5)	1.27(8)	0.66(3)	0.44(16)
UKQCD[15]	0.67($\frac{7}{8}$)	1.01($\frac{30}{13}$)	0.70($\frac{7}{10}$)	0.66($\frac{10}{15}$)

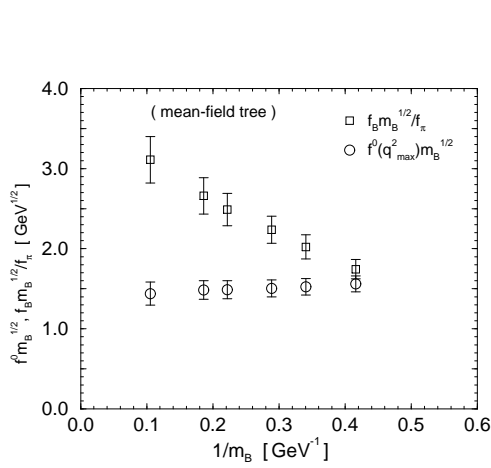


Figure 14. Comparison of $f^0(q^2_{max})$ with f_B/f_π at mean-field tree level.

factors at $q^2 = 0$ are listed in Table 3.3.2.

4. Summary

In summary, the heavy-light decay constant in the quenched approximation now has a precision

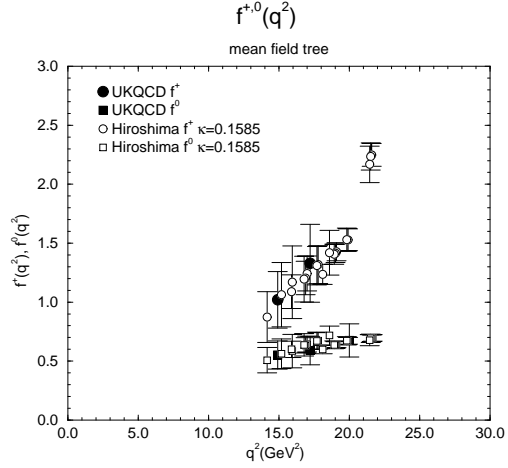


Figure 15. q^2 dependence of form factors.

of less than 20% once one uses the appropriate scale setting. For higher precision of less than 10%, full one loop renormalization including the operator mixing will be important. Comparison with decay constants with NRQCD+clover will be another consistency check of the result.

A lot of more work with higher statistics as

well as use of improved action are needed for the semileptonic form factors. Reducing the systematic errors are especially important to understand the q^2 dependence and the puzzling violation of the soft pion theorem. Studies of $q^2 \sim q_{max}^2$ are important, because the future experiments such as KEK B factory which will start in 1999 are expected to have the luminosity of about $10^{34} cm^{-2} s^{-1}$ producing 10^8 $B \bar{B}$ pairs after 3-4 years of running. which is 30 times more than the CLEO data. One may expect to see enough experimental data even for the small recoil region.

Studies at the scaling behavior at $q^2 = 0$ around charm quark mass range to test the LCSR with higher statistics are also very important.

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